

# Ratio of bulk to shear viscosity in a quasigluon plasma: from weak to strong coupling

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## Abstract

The ratio of bulk to shear viscosity is expected to exhibit a different behaviour in weakly and in strongly coupled systems. This can be expressed by its dependence on the squared sound velocity. In the high temperature QCD plasma at small running coupling, the viscosity ratio is uniquely determined by a quadratic dependence on the conformality measure, whereas in certain strongly coupled and nearly conformal theories this dependence is linear. Employing an effective kinetic theory of quasiparticle excitations with medium-modified dispersion relation, we analyze the ratio of bulk to shear viscosity of the gluon plasma. We show that in this approach, depending on the temperature, the viscosity ratio exhibits either of these dependencies found by means of weak coupling perturbative or strong coupling holographic techniques. The turning point between the two different dependencies is located around the maximum in the scaled interaction measure.

*Keywords:* gluon plasma, bulk viscosity, shear viscosity, quasiparticle model, effective kinetic theory

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## 1. Introduction

The observation that the quark-gluon plasma (QGP) formed in ultra-relativistic heavy-ion collisions at the Relativistic Heavy Ion Collider (RHIC) behaves almost like a perfect fluid [1, 2], inspired numerous theoretical efforts to quantify the transport properties of the produced strongly interacting medium. In particular, various attempts have been proposed to calculate bulk and shear viscosities for QCD matter by means of different techniques (see [3] for a recent review). Likewise endeavours have been made to reliably extract information on the viscosity coefficients from RHIC measurements by continuously improving the phenomenological analysis of data (see [4] for a recent review). While currently heavy-ion collision phenomenology seems to focus on the shear viscosity as a relevant quantity, a firm knowledge of the bulk viscosity as well as other transport coefficients is equally needed to characterize the properties of the QGP concisely.

The bulk viscosity  $\zeta$  is a particular transport coefficient vanishing, within many models, in the non-relativistic and ultra-relativistic limits as well as for conformally invariant systems. In strongly interacting matter, the bulk viscosity to entropy density ratio  $\zeta/s$  is expected to be small outside the deconfinement and chiral phase transition region. In the vicinity of the transition region, however, it is expected to become large and even to diverge at a second order phase transition [5, 6, 7, 8, 9, 10]. Therefore, a precise knowledge of  $\zeta$  might be essential to understand and quantify the transport dynamics of the strongly interacting fluid in the critical region.

The shear viscosity to entropy density ratio  $\eta/s$  for QCD matter, on the other hand, is expected to be large, i.e.  $\eta/s > 1$ , at large temperatures  $T$ , whereas in the vicinity of the deconfinement transition temperature  $T_c$  it exhibits a minimum. Such a behaviour is known for a variety of liquids and gases [11, 12] and was conjectured [12, 13] to appear also in strongly interacting matter.

The ratio  $\eta/s$  was shown to exhibit a universal behaviour in gauge theory plasmas at infinite 't Hooft coupling and infinite number of colours [11, 14], which led to the conjecture of a fundamental lower bound  $(\eta/s)_{KSS} \geq 1/(4\pi)$  (Kovtun-Son-Starinets bound in natural units) for any physical system [11, 15]. A similar universal behaviour for the bulk viscosity or other transport coefficients is in contrast not known [16, 17]. Nonetheless, in specific strongly coupled and nearly conformal theories that allow for a holographically dual supergravity description [18, 19], the ratio of bulk to shear viscosity was

found to behave as

$$\zeta/\eta \sim \left( \frac{1}{3} - v_s^2 \right), \quad (1)$$

where the proportionality constant is of order  $\mathcal{O}(1)$  and  $v_s^2 = \partial P / \partial \epsilon$  is the squared sound velocity expressed as derivative of the thermal pressure  $P$  with respect to the energy density  $\epsilon$ . Recently, a lower bound on this ratio,  $(\zeta/\eta)_B \geq 2(1/k - v_s^2)$  (Buchel bound), was conjectured for strongly coupled gauge theories in  $k$  spatial dimensions as being valid for all temperatures, where the holographically dual supergravity description is allowed [19]. This bound is exactly saturated as kinematical identity for all strongly coupled theories with holographic dual related to non-conformal branes [20].

In contrast, in scalar field theory [21] as well as for photons interacting with massive particles of a thermal fluid [22], it was shown that the ratio  $\zeta/\eta$  is uniquely determined by

$$\zeta/\eta = 15 \left( \frac{1}{3} - v_s^2 \right)^2. \quad (2)$$

In QCD, bulk and shear viscosities were calculated for large temperatures and small running coupling within kinetic theory [23, 24]. Under the relaxation time approximation by assuming equal collision rates for bulk and shear viscosities [23], these results give rise to a ratio  $\zeta/\eta$  that behaves parametrically like in Eq. (2). Thus, in perturbative QCD (pQCD) the bulk to shear viscosity ratio depends quadratically on the conformality measure  $\Delta v_s^2 = (1/3 - v_s^2)$ . This implies that the Buchel bound is violated in high temperature QCD at small running coupling.

Even though the precise holographic dual to QCD is currently unknown, one could expect from Eqs. (1) and (2) that for deconfined strongly interacting matter the ratio  $\zeta/\eta$  undergoes a gradual change from the non-perturbative to the perturbative regime, rendering its dependence on the conformality measure from a linear to a quadratic one. It is therefore interesting to study the temperature dependence of the viscosity ratio from  $T_c$  towards large  $T$  in order to understand how the transport properties of the medium are influenced by its thermal properties.

In this work, considering the gluon plasma as composed of quasiparticle excitations with medium-modified dispersion relation, we analyze the properties of the bulk to shear viscosity ratio. The equilibrium thermodynamics of such a quasiparticle model (QPM) was shown to describe successfully lattice

QCD results on the equation of state and related quantities for pure  $SU_c(3)$  gauge theory [25, 26]. To study the transport properties of the quasigluon plasma, we apply an effective kinetic theory approach [27]. From the results for bulk and shear viscosities obtained in this way, we show that in a quasi-particle picture the ratio  $\zeta/\eta$  exhibits indeed the above discussed behaviour with  $\Delta v_s^2$ : At large temperatures, i.e. in the perturbative regime, the dominant dependence of  $\zeta/\eta$  on the conformality measure is quadratic, whereas near the deconfinement transition it renders into a linear behaviour. The turning point between the two is located in the vicinity of the maximum in the scaled interaction measure. Thus, on the level of transport properties, a phenomenological quasiparticle model of the gluon plasma provides a systematic interpolation between the regimes of weak and strong coupling.

## 2. Effective Kinetic Theory

Assuming that the kinetics of quasigluons is appropriately described by an effective kinetic equation of Boltzmann-Vlasov type, the phase-space behaviour of the single-particle distribution function  $f$  follows from  $(\mathcal{L} + \mathcal{V})f = \mathcal{C}[f]$ , where  $\mathcal{L}$  is the Liouville operator,  $\mathcal{V}$  is the Vlasov mean field term and  $\mathcal{C}$  is the collision term. In local thermal equilibrium, the functional  $\mathcal{C} = 0$ , and the thermodynamics of the QPM should be recovered [25, 28]. This is accomplished by  $f \rightarrow n(T) = d(\exp(E^0/T) - 1)^{-1}$ , where  $n(T)$  is the Bose distribution function for gluon excitations with energy  $E^0 = \sqrt{\vec{p}^2 + \Pi(T)}$  and  $\Pi(T)$  is the temperature dependent (but momentum independent) gluon self-energy. The degeneracy factor for colour and polarization degrees of freedom is  $d = 16$ . Self-consistency of this approach dictates the form of the Vlasov term. In particular, this implies that in local thermal equilibrium  $\mathcal{V}$  must be related to the temperature dependence of the self-energy in order to maintain thermodynamic self-consistency [29, 30].

## 3. Ratio of Bulk to Shear Viscosity

The viscosity coefficients can be derived from the above effective kinetic theory by assuming small deviations from local thermal equilibrium. The expressions for bulk and shear viscosities obtained within relaxation time approximation read [29, 30, 31]

$$\eta(T) = \frac{1}{15T} \int \frac{d^3\vec{p}}{(2\pi)^3} n(T) [1 + d^{-1}n(T)] \frac{\tau}{(E^0)^2} \vec{p}^4, \quad (3)$$

$$\begin{aligned} \zeta(T) = & \frac{1}{T} \int \frac{d^3 \vec{p}}{(2\pi)^3} n(T) [1 + d^{-1} n(T)] \frac{\tau}{(E^0)^2} \\ & \times \left\{ [(E^0)^2 - a] v_s^2(T) - \frac{1}{3} \vec{p}^2 \right\}^2, \end{aligned} \quad (4)$$

where  $\tau$  denotes the relaxation time, which is related to the collision term  $\mathcal{C}$ , and  $a = T^2(\partial\Pi(T)/\partial T^2)$ .

To quantify the transport coefficients for the quasigluon plasma from Eqs. (3) and (4), we use for the gluon self-energy [25]

$$\Pi(T) = \frac{1}{2} T^2 G^2(T), \quad (5)$$

where  $G^2(T)$  is the temperature dependent effective coupling,

$$G^2(T) = \frac{16\pi^2}{11 \ln [\lambda(T - T_s)/T_c]^2}, \quad (6)$$

defined such that at high temperatures the expressions for bulk thermodynamic quantities of  $SU_c(3)$  gauge theory are recovered by the QPM. With the parameters  $T_s/T_c = 0.73$  and  $\lambda = 4.3$ , cf. [30], pure  $SU_c(3)$  lattice gauge theory results for the equation of state [32, 33] are nicely described. The quality of this description is quantified by  $\chi^2/\text{d.o.f.} = 1.4 \cdot 10^{-2}$  for the lattice data from [32] and by  $\chi^2/\text{d.o.f.} = 1.5 \cdot 10^{-3}$  for the lattice data from [33]. Within this model, the leading terms of the squared sound velocity at asymptotically high temperatures, where  $1 \gg G^2 \gg |T(dG^2/dT)|$  holds, read [23]

$$v_s^2(T) = \frac{1}{3} + \frac{5}{48\pi^2} T \frac{dG^2(T)}{dT} + \mathcal{O} \left( G^2(T) T \frac{dG^2(T)}{dT} \right), \quad (7)$$

which implies that  $v_s^2$  reaches  $1/3$  only for asymptotically large  $T$ .

From Eqs. (3) and (4) and by assuming an equal relaxation time for both transport coefficients, the ratio of bulk to shear viscosity in a quasiparticle description of the gluon plasma may be written in the form

$$\begin{aligned} \frac{\zeta}{\eta} = & 15 (\Delta v_s^2)^2 - 30 \Delta v_s^2 [\Pi(T) - a] v_s^2(T) \frac{\mathcal{I}_0(T)}{\mathcal{I}_{-2}(T)} \\ & + 15 [\Pi(T) - a]^2 (v_s^2(T))^2 \frac{\mathcal{I}_2(T)}{\mathcal{I}_{-2}(T)}, \end{aligned} \quad (8)$$

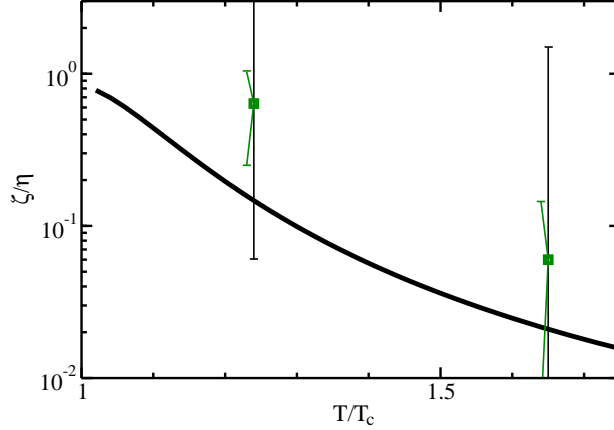


Figure 1: Scaled temperature dependence of the bulk to shear viscosity ratio Eq. (8) (solid curve) for  $1.02 \leq T/T_c \leq 1.75$  compared with available lattice QCD results from [34] (squares). The displaced error bars are determined from the statistical errors in  $\zeta$  and  $\eta$  [34]. The non-displaced error bars are obtained from the conservative upper and lower bounds on  $\zeta$  given in [34] and the statistical errors in  $\eta$ .

with momentum integrals

$$\mathcal{I}_k(T) = \int \frac{d^3\vec{p}}{(2\pi)^3} n(T) [1 + d^{-1}n(T)] \frac{\tau}{(E^0)^2} \vec{p}^{2-k}. \quad (9)$$

The momentum integrals receive dominant contributions from thermal quasiparticle momenta,  $|\vec{p}| \sim T$ , with proportionality constant of order  $\mathcal{O}(1)$ . They follow, for all  $T \geq T_c$ , the hierarchy  $\mathcal{I}_{-2}/T^2 \gg \mathcal{I}_0 \gg T^2\mathcal{I}_2 > 0$ . The terms in Eq. (8), which are proportional to  $[\Pi(T) - a]$ , contain only the temperature derivative of the effective coupling  $G^2(T)$ , since the leading  $T$ -dependence cancels out in this combination. They would, therefore, not be present in the case of a temperature independent coupling  $g$ , i.e. in models with a quasiparticle mass  $M(T)$  with trivial temperature dependence of the form  $M(T) \sim Tg$ . Moreover, for constant  $M$  one finds Eq. (8) for the ratio  $\zeta/\eta$ , but with  $[\Pi(T) - a]$  replaced by  $M^2$  and  $\Delta v_s^2 = 0$  [25, 30].

The ratio  $\zeta/\eta$  from Eq. (8) is quantified in Fig. 1 as a function of  $T/T_c$  for a gluon plasma described by quasiparticle excitations with medium-modified dispersion relation and a momentum independent relaxation time  $\tau$ . For comparison, also shown in Fig. 1 is the bulk to shear viscosity ratio obtained from lattice QCD results in [34]. The bulk and shear viscosities calculated in lattice QCD are fairly well reproduced within our quasiparticle model [30].

Noticing the large errors in Fig. 1, the temperature dependence of the viscosity ratio in the quasigluon plasma is seen in Fig. 1 to be as well consistent with that found from the lattice gauge theory calculations.

For  $T \gtrsim 1.5 T_c$ , the viscosity ratio is entirely determined by the pQCD-like behaviour from Eq. (2). However, for  $T$  near  $T_c$ , the non-perturbative contributions in Eq. (8) become significant, resulting in a reduction of the  $\zeta/\eta$  ratio. As evident from Fig. 1, for  $T \gtrsim 1.02 T_c$ , the bulk viscosity is smaller than the shear viscosity in the quasigluon plasma. For  $T \rightarrow 1.02 T_c$ , the ratio  $\zeta/\eta$  increases and reaches about 0.78. Quite similar values for this ratio were reported in holographic approaches [35, 36, 37] and used in hydrodynamic simulations when studying the influence of a finite  $\zeta$  and  $\eta$  on the elliptic flow [38]. Nonetheless, we note that in a different holographic approach [19] a much larger value for  $\zeta/\eta$  in this temperature region was predicted.

The viscosity ratio  $\zeta/\eta$  in Eq. (8) suggests apparently both, a quadratic behaviour  $\propto (\Delta v_s^2)^2$ , as found in the regime of small QCD running coupling [23], and a linear behaviour  $\propto \Delta v_s^2$ , which is in line with the dependence found in specific strongly coupled systems near conformality [16, 18, 19]. A similar combination of dependencies was also pointed out in [18, 35] as phenomenologically relevant relation. One should emphasize, however, that all other quantities entering the  $\zeta/\eta$  ratio in Eq. (8) depend on  $T = T(\Delta v_s^2)$ . Therefore, Eq. (8) cannot be understood as a strict expansion series in powers of the conformality measure. Rather, for a quantification any temperature dependence has to be converted into a  $\Delta v_s^2$ -dependence.

The behaviour of  $T$  with  $\Delta v_s^2$  can be obtained by inverting the relation between the squared sound velocity  $v_s^2(T)$  and the temperature. In the considered quasigluon plasma,  $v_s^2(T)$  follows as

$$v_s^2(T) = \frac{1}{3} \frac{1}{\left[1 + \frac{T}{3} \frac{\mathcal{J}_1(T)}{\mathcal{J}_2(T)} \frac{d}{dT} \left( \frac{\Pi(T)}{T^2} \right) \right]}, \quad (10)$$

where the entering integrals read

$$\begin{aligned} \mathcal{J}_1 = \int_0^\infty dx \frac{x^2}{(e^{\sqrt{x^2+z}} - 1)} & \left( \frac{1}{\sqrt{x^2+z}} - \frac{(\frac{4}{3}x^2+z)}{2\sqrt{x^2+z}^3} \right. \\ & \left. - \frac{(\frac{4}{3}x^2+z) e^{\sqrt{x^2+z}}}{2(x^2+z) (e^{\sqrt{x^2+z}} - 1)} \right) \end{aligned}$$

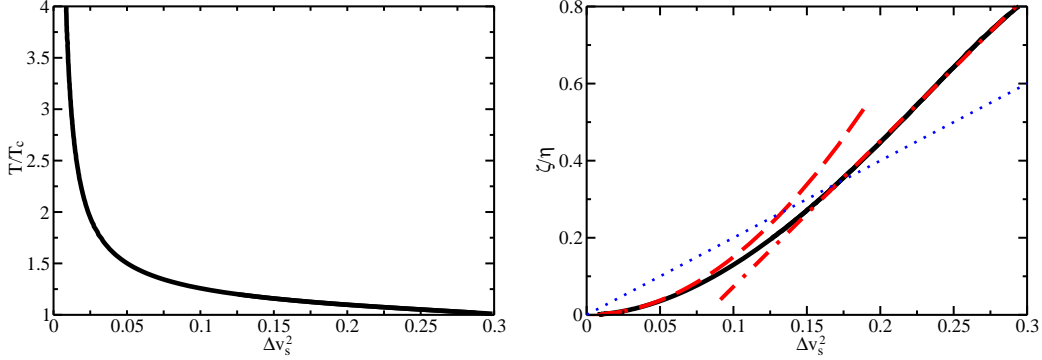


Figure 2: Left: Scaled temperature dependence of  $\Delta v_s^2$  in the QPM. Right: The ratio  $\zeta/\eta$  from Eq. (8) (solid curve) as a function of the conformality measure  $\Delta v_s^2$ . The dash-dotted curve shows a linear fit  $\zeta/\eta = \alpha \Delta v_s^2 + \beta$  with  $\alpha = 3.78$  and  $\beta = -0.305$ . The dashed curve depicts the quadratic contribution in Eq. (8),  $15 (\Delta v_s^2)^2$ . The dotted curve shows Buchel's conjectured lower bound for  $\zeta/\eta$ .

and

$$\mathcal{J}_2 = \int_0^\infty dx \frac{x^2 \left( \frac{4}{3}x^2 + z \right)}{(e^{\sqrt{x^2+z}} - 1) \sqrt{x^2+z}}$$

with  $z = \Pi(T)/T^2$ . For asymptotically large  $T$ , Eq. (10) simplifies to Eq. (7).

The relation  $T = T(\Delta v_s^2)$  following from Eq. (10) is shown in the left panel of Fig. 2. The right panel of this figure quantifies the resulting dependence of the ratio  $\zeta/\eta$  on the conformality measure  $\Delta v_s^2$  in the interval  $0.01 \leq \Delta v_s^2 \leq 0.3$ . For small values of  $\Delta v_s^2$ , that is for large temperatures  $T \gtrsim 1.5 T_c$  (see left panel of Fig. 2), the viscosity ratio shows a quadratic behaviour with  $\Delta v_s^2$ . Indeed, for asymptotically large  $T$ , one finds with Eq. (7), that  $\Delta v_s^2 \simeq -5/(24\pi^2)T^2(dG^2/dT^2)$ , where  $|T^2(dG^2/dT^2)| \ll 1$ . Then, all terms in Eq. (8) are at leading order in  $G^2$  proportional to  $(T^2 dG^2/dT^2)^2$ , which implies in turn, that in this temperature regime all terms in Eq. (8) are at leading order proportional to  $(\Delta v_s^2)^2$ . However, since  $T^4 \mathcal{I}_2/\mathcal{I}_{-2} \ll T^2 \mathcal{I}_0/\mathcal{I}_{-2} \ll 1$ , the numerically dominant contribution to  $\zeta/\eta$  in Eq. (8) is given by  $15 (\Delta v_s^2)^2$ , and Eq. (2) is recovered.

With increasing  $\Delta v_s^2$ , i.e. for decreasing  $T$  (see left panel of Fig. 2), the bulk to shear viscosity ratio renders numerically into a linear rise with  $\Delta v_s^2$  as seen in the right panel of Fig. 2. In fact, with decreasing  $T$  towards  $T_c$ , the quadratic term  $15 (\Delta v_s^2)^2$  in Eq. (8) is cancelled to a large degree of accuracy by the other terms, which in the regime  $0.17 \lesssim \Delta v_s^2 \leq 0.3$  sum up



to approximately  $-15 (\Delta v_s^2)^2 + \alpha \Delta v_s^2 + \beta$  with constant coefficients  $\alpha$  and  $\beta$ . Thus, a linear relation  $\zeta/\eta = \alpha \Delta v_s^2 + \beta$  emerges as evident from Fig. 2 (right panel). The onset of this change in the behaviour with the conformality measure is located around  $T \simeq 1.15 T_c$ . This lies in the direct proximity of the maximum in the scaled interaction measure [30].

Fig. 2 (right panel) depicts, in addition, Buchel's lower bound on the ratio  $\zeta/\eta$  [19] (dotted curve). In our quasiparticle description of the gluon plasma this bound is satisfied also only for temperatures  $T \lesssim 1.15 T_c$ , when  $\Delta v_s^2 \gtrsim 0.17$ . We note, however, that this observation is based on the used approximation of a momentum independent relaxation time, which is the same for both viscosity coefficients. Therefore, it would be interesting to study how a momentum dependent  $\tau$  in Eq. (8), as proposed in [31], would influence the quantitative comparison of Eq. (8) with the conjectured lower bound and the quantitative dependence of the viscosity ratio on  $\Delta v_s^2$ .

This demonstrates that within an effective kinetic theory approach for quasigluon excitations, which is supplemented by information from equilibrium thermodynamics, both limiting behaviours of the bulk to shear viscosity ratio with  $\Delta v_s^2$  known from the weak [23] and strong coupling regimes [16, 18, 19] can be found. Thus, the quasiparticle model for the gluon plasma provides a phenomenological method to describe non-perturbative effects. Within such a model, one can effectively describe thermodynamics as well as transport properties of the gluon plasma in the whole temperature range from asymptotically large temperatures towards the critical region just above deconfinement.

#### 4. Conclusion

Within our quasiparticle model for the gluon plasma, which is based on an effective kinetic theory, we calculated explicitly the bulk to shear viscosity ratio  $\zeta/\eta$  in the relaxation time approximation. We have shown, that this ratio exhibits, depending on  $T$ , a quadratic or a linear behaviour with the conformality measure  $\Delta v_s^2$  of the medium. Similar dependencies are known from small running coupling QCD [23] and holographic [18, 19] approaches, respectively. While at asymptotically large  $T$  the quadratic dependence on  $\Delta v_s^2$  is dominant in our model, the linear dependence becomes more important when approaching  $T_c$ , governing effectively the qualitative behaviour of the viscosity ratio for  $1.02 \lesssim T/T_c \lesssim 1.15$ . The turning point in the behaviour is located around the maximum in the scaled interaction measure. Therefore,

besides a proper description of equilibrium thermodynamics of pure  $SU_c(3)$  lattice gauge theory, our quasiparticle model of the gluon plasma also describes its viscosity coefficients, cf. [30], and reproduces the dependencies of their ratio on  $\Delta v_s^2$  as expected in the weak and strong coupling regimes, thus, providing a systematic link between the two. For momentum independent relaxation times, assumed to be common for bulk and shear viscosities, we find a significant dependence of  $\zeta/\eta$  on  $v_s^2$ . This might imply consequences for viscous hydrodynamic simulations, cf. the studies in [38, 39], in particular for LHC energies, where a larger temperature range is probed. However, for any phenomenological application one would have to include quark degrees of freedom into the model calculations.

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